Differential Evolution as Applied to Electromagnetics

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Abstract

In electromagnetics, optimization problems generally require high computational resources and involve a large number of unknowns. They are usually characterized by non-convex functionals and continuous spaces suitable for strategies based on Differential Evolution (*DE*). In such a framework, this paper is aimed at presenting an overview of Differential Evolution-based approaches used in electromagnetics, pointing out novelties and customizations with respect to other fields of application. Starting from a general description of the evolutionary mechanism of Differential Evolution, Differential Evolution-based techniques for electromagnetic optimization are presented. Some hints on the convergence properties and the sensitivity to control parameters are also given. Finally, a comprehensive coverage of different Differential Evolution formulations in solving optimization problems in the area of computational electromagnetics is presented, focusing on antenna synthesis and inverse scattering.

Keywords: Differential evolution; optimization methods; antenna arrays; antennas; inverse problems; electromagnetic scattering inverse problems

1. Introduction

In the last decades, the introduction and use of stochastic-based optimization algorithms has had a non-negligible impact on several areas of research. It has also contributed to the development of new applications and industrial processes. Among such algorithms, evolutionary algorithms (EAs) have received the widest diffusion because of their attractive features as global optimizers [1-5]. As a matter of fact, they have been shown to effectively deal with complex functionals, despite their simple implementation and use [6], thanks to the reduced number of control parameters to be set. Furthermore, the main drawback (i.e., the convergence rate) limiting their applicability has been further mitigated by exploiting their parallelism, thanks to modern personal computers and clusters.

The Differential Evolution (DE) algorithm was first proposed by Storn and Price [7] as a population-based evolutionary algorithm for the optimization of continuous variables in multi-dimensional spaces. Like genetic algorithms (GAs) [8], Differential Evolution is modeled on the competitive mechanisms of natural selection and genetic pressure studied by Darwin, and successively adapted to the solution of artificial problems by Holland [9]. The evolutionary mechanism of Differential Evolution exploits the same evolutionary operators as genetic algorithms, but they are executed in a different order. More specifically, mutation and crossover modify the parameter vectors before selection and not vice versa, as for genetic algorithms. Accordingly, the "destructive" effect of mutation in genetic algorithms is avoided, since it is performed at the beginning of each generation loop and not at the end. Moreover, both the best and the average fitness values increase/decrease monotonically (without the need of ad hoc operators, namely, elitism) since the competition between parents and children (i.e., the selection) takes place after crossover. Furthermore, an effective sampling of the solution space is also insured, since the whole population of trial solutions is used as the mating pool, without giving advantage to the fittest individuals, and the mutant vectors are generated by using other individuals randomly chosen from the population.

Although much attention has been devoted to other evolutionary algorithms (e.g., the real-coded genetic algorithm (RGA) [10-13] or the Particle Swarm Optimizer (PSO) [14-18]) to deal with the optimization of floating-point parameters, more recently Differential Evolution has been effectively used, as confirmed by the increasing growth in various research papers with regard to Differential Evolution (Figure 1) and to Differential Evolution in electromagnetics (Figure 2).

This paper presents a survey of Differential Evolution as applied to the solution of electromagnetic problems, as has been done for the genetic algorithm [19, 20] and the Particle Swarm Optimizer [21], to give a comprehensive report of the latest progress in this field. The outline of the paper is as follows. A simple and standard version of Differential Evolution is described in Section 2. However, since electromagnetic optimization generally involves a large set of parameters and high computational resources, customized versions of Differential Evolution are carefully reviewed (Section 2.2). Some hints on the mathematical issues and some guidelines for the choice of the control parameters

 ISSN 1045-9243/2011/$25©2011 IEEE

in electromagnetic applications are given in Section 3. Section 4 presents a comprehensive overview of the application of Differential Evolution in electromagnetics, ranging from the synthesis of antenna elements and arrays (Section 4.1) and the solution of inverse-scattering problems (Section 4.2) up to the design of microwave components (Section 4.3). Eventually, some conclusions are drawn (Section 5).

2. Differential Evolution Strategy

Differential Evolution is aimed at evolving a population of S trial solutions to achieve the optimal (global) solution of the optimization problem at hand. Each individual is identified by a chromosome that codes a set of unknowns descriptive of the problem solution. Let us consider the optimization of N real-coded parameters, x_n, $n = 1,...,N$, within a search space with lower (i.e., x_n^{min}, $n = 1,...,N$) and/or upper bounds (i.e., x_n^{max}, $n = 1,...,N$). The exploration of the solution space can also be subjected to additional constraints, mathematically expressed through either equalities, $l_i(\underline{x}) = 0$, $i = 1,...,I$, or inequalities, $f_j(\underline{x}) \leq 0$, $j = 1,...,J$.

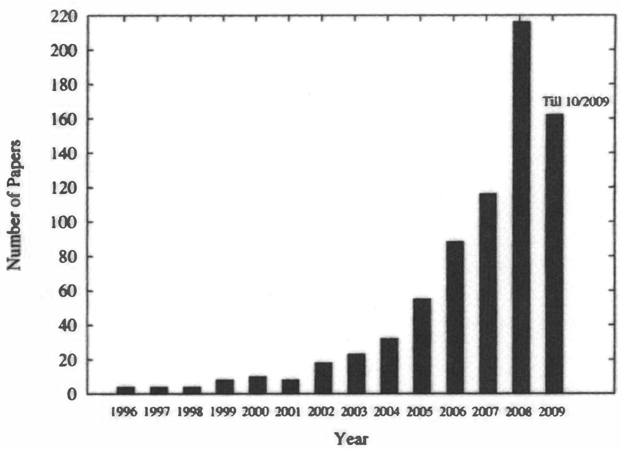

Figure 1. The number of Differential-Evolution-related papers published each year in all research fields (based on IEEE/IEE databases).

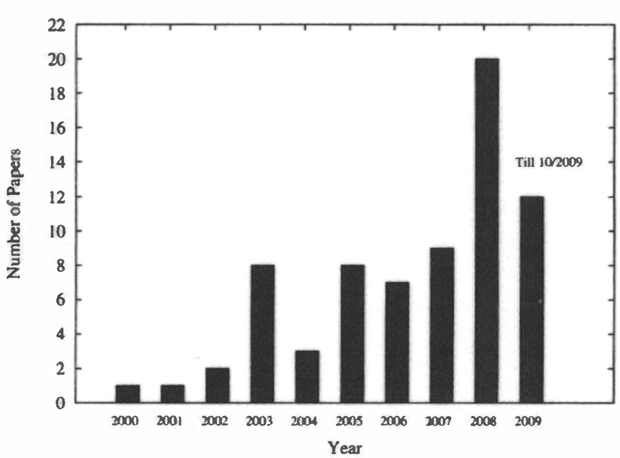

Figure 2. The number of Differential-Evolution-related papers published each year in electromagnetics (based on IEEE/IEE databases).

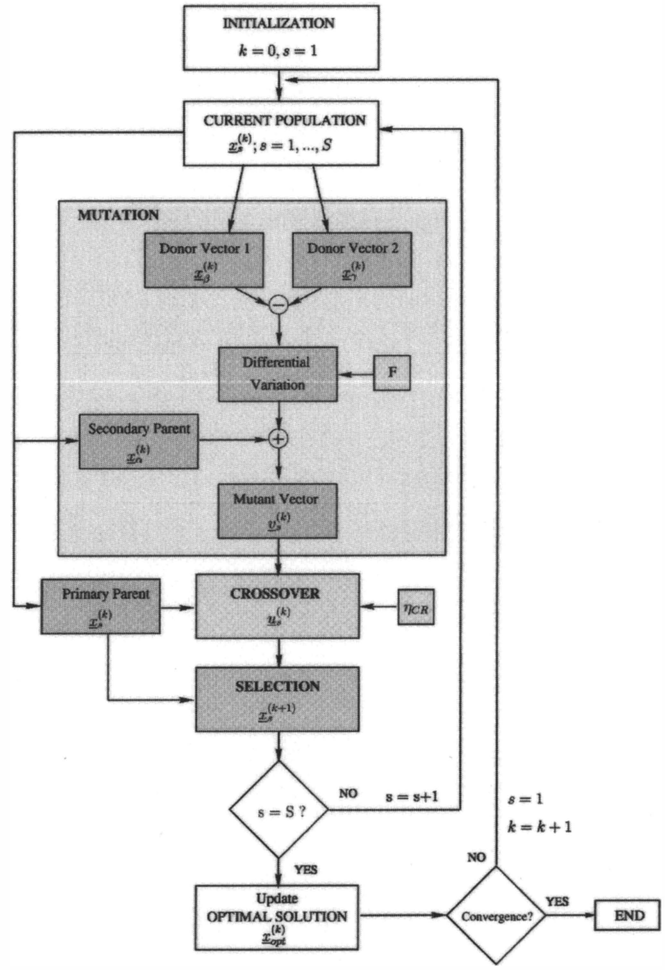

Figure 3. A conventional Differential Evolution flowchart.

The cost functional to be optimized can be represented by a single function, $\Psi(\underline{x})$, or by more functions, $\Psi_t(\underline{x})$, $t = 1,2,...,T$, as in multi-objective optimization [22].

The iterative procedure implemented by Differential Evolution is shown in Figure 3. At initialization, the trial solutions, $\underline{x}_s^{(0)}$, $s = 1,...,S$, are usually chosen by sampling the parameter space with a uniform probability according to the relationship $x_{n,s} = r(n,s)x_{n,s}^{max} + [1 - r(n,s)]x_{n,s}^{min}$ [23]. $r(n,s)$ is a stochastic variable uniformly distributed within $r \in [0,1]$ and varying with the indices n and s, or is stochastically generated around a guessed solution with a normal distribution [24, 25]. As far as the reproduction loop of Differential Evolution is concerned, both conventional and modified versions of Differential Evolution have been used when dealing with electromagnetics.

2.1 Conventional Differential Evolution

In Differential Evolution, new solutions are basically generated by adding weighted real vectors (i.e., differential variations), computed as the difference from couples of other individuals taken from the population [7]. The new solutions (i.e., the children) are maintained in the successive population if they outperform the corresponding parents (i.e., if their fitness is better). More specifically, the reproduction loop is performed in classic Differential Evolution as shown in Figure 3. The genetic operators are executed in the following order: first mutation, then crossover, and finally selection.

The main novelty of Differential Evolution with respect to genetic algorithms is mutation. For each individual of the kth current population, $\underline{x}_s^{(k)}$, $s = 1,\ldots,S$, k being the iteration index, a *mutant vector*, $\underline{v}_s^{(k)}$, is generated as follows:

$$\underline{v}_s^{(k)} = \underline{x}_\alpha^{(k)} + F\sum_y \left[\underline{x}_{\beta_y}^{(k)} - \underline{x}_{\gamma_y}^{(k)}\right],\ s = 1,\ldots,S, \quad (1)$$

where y is the number of differential variations, and α, β_y, and γ_y can assume either the index of the best solution or be randomly chosen from the population ($\beta_y \neq \gamma_y$). Moreover, $F \in (0,2]$ is the scaling factor. The vectors $\underline{x}_s^{(k)}$ and $\underline{x}_\alpha^{(k)}$ are referred to as the *primary parent* and *secondary parent*, respectively. $\underline{x}_{\beta_y}^{(k)}$ and $\underline{x}_{\gamma_y}^{(k)}$ are called *donor vectors*.

It is simple to note (Equation (1)) that the differential variations define the "direction" of the mutation within the solution space, while the search step is properly scaled/amplified by F. As a result, the differential mutation operates like a local search in those regions of the parameter space individualized by primary parents. The choice of the primary parents (e.g., $\alpha = s$ or $\alpha = best$ or $\alpha = random$) therefore has non-negligible effects on the behavior/evolution of the search procedure. Improved strategies, adapting the mutation strategy during the iterative process, have been proposed [26, 27] to combine both exploration and exploitation capabilities.

As far as the crossover is concerned, it is applied to the *primary parent*, $\underline{x}_s^{(k)}$, and to the mutant vector, $\underline{v}_s^{(k)}$. In the literature, two different versions of crossover have mainly been used: the binomial crossover [28] and the exponential crossover [7]. The former operates as follows:

$$u_{n,s}^{(k)} = \begin{cases} v_{n,s}^{(k)} & \text{if } r < \eta_{CR} \\ x_{n,s}^{(k)} & \text{otherwise} \end{cases},\ n = 1,\ldots,N, \quad (2)$$

where $\eta_{CR} \in (0,1]$ is an input parameter. Otherwise, the exponential crossover is based on a slightly different mechanism:

$$u_{n,s}^{(k)} = \begin{cases} v_{n,s}^{(k)} & \text{if } L_1 \leq n \leq L_2 \\ x_{n,s}^{(k)} & \text{otherwise} \end{cases},\ n = 1,\ldots,N, \quad (3)$$

where L_1, L_2 are random integer numbers chosen in the range $1 \leq L_1 \leq L_2 \leq N$.

The selection operator works deterministically, discarding the worst individual between $\underline{u}_s^{(k)}$ and $\underline{x}_s^{(k)}$. In a minimization problem, it turns out that

$$\underline{x}_s^{(k+1)} = \begin{cases} \underline{u}_s^{(k)} & \text{if } \Psi\left[\underline{u}_s^{(k)}\right] \leq \Psi\left[\underline{x}_s^{(k)}\right] \\ \underline{x}_s^{(k)} & \text{otherwise} \end{cases}. \quad (4)$$

As a notation, the acronym $DE/\alpha/y/z$ [28] has been introduced to identify Differential Evolution variants, where α, y, and z denote the choice of the secondary parent, the number of differential variations in Equation (1), and the crossover method, respectively.

2.2 Modified Differential Evolution Approaches

Optimization problems arising in electromagnetics generally involve a large number of parameters, and they are characterized by high dimensionality. Although Differential Evolution outperforms other evolutionary algorithms when dealing with a small/limited number of unknowns (e.g., genetic algorithms [29] or Particle Swarm Optimizer variants [18]), it has still shown low convergence properties or difficulties in achieving the global best solution [30-32]. To properly deal with these drawbacks or characteristics, improved versions of Differential Evolution have been proposed. In the following, some representative examples are reported, and their innovative aspects are pointed out.

2.2.1 Differential Evolution with Individuals in Group

In [33], an innovative Differential-Evolution-based approach was proposed to deal with the reconstruction of multiple perfectly conducting objects: more specifically, the retrieval of their locations and contours, as well as their number. As expected, knowledge of the number of cylinders lying within the investigation domain can greatly improve the efficiency of the inversion procedure and can also guarantee more-accurate reconstructions [34], but this is not always a priori available. However, the Differential Evolution strategy with individuals in groups (GDES) considered in [33] proved its efficacy without such information. The population of the Differential Evolution strategy with individuals in groups is partitioned into several groups, where the individuals of each group code the same number of cylinders. Mutation, crossover, and selection are independently applied to each group, since the length of the chromosomes (i.e., the number of unknowns) is different for each cluster. An additional operator, the *group competition operator*, was then introduced to identify the correct number of cylinders.

At the initialization ($k = 0$), the number of trial solutions of each group, $S_g^{(0)}$, $g = 1,\ldots,G$, with G being the number of groups, is determined on the basis of the chromosome length of the corresponding individuals [33], as follows:

$$S_g^{(0)} = \frac{2gS}{G(G+1)}. \quad (5)$$

Successively, the dimensions of the groups are determined on the basis of the average fitness values of their members. To keep diversity among the individuals of the population, empirical thresholds have been considered. The group dimension is updated according to the following relationship:

$$S_g^{(k)} = \max\left\{3, \frac{S_g^{(0)}}{2}, S\frac{\left[1-\frac{\overline{\Psi}_g^{(k-1)}}{\sum_{g=1}^{G}\overline{\Psi}_g^{(k-1)}}\right]}{G-1}\right\}, \qquad (6)$$

with $\overline{\Psi}_g^{(k)}$ being the average fitness value of the gth group at the kth iteration.

2.2.2 Dynamic Differential Evolution

To enhance the convergence properties and to improve the ability of the algorithm to quickly adapt itself to the evolution of the optimization process, as well as to the landscape of the cost function to be maximized/minimized, a dynamic Differential Evolution (DDE) strategy was proposed in [35]. The newborn children compete immediately with the parent for the possibility to survive throughout the evolution. Moreover, the optimal solution is instantaneously updated whenever the fitness of a new individual is better than that of the current optimal solution. Although based on the DE/best/1/bin approach, the evolutionary mechanism of the dynamic Differential Evolution strategy [35] differs from that of the conventional Differential Evolution, as detailed in Figure 4:

- Step 1 – *Mutation*: A mutant vector is defined for the sth individual as

  $$v_{n,s}^{(k)} = x_{n,opt}^{(k)} + F_n\left[x_{n,\beta}^{(k)} - x_{n,\gamma}^{(k)}\right],\ n = 1,\ldots,N, s = 1,\ldots,S, \qquad (7)$$

  where F_n is a random number uniformly distributed in the range $[0,1]$ and $\beta,\gamma \in [1,S]$ with $\beta \neq \gamma \neq s$;

- Step 2 – *Crossover*: After mutation, the binomial crossover (Equation (2)) is used to generate a new child, $\underline{u}_s^{(k)}$;

- Step 3 –*Solution Update*: A parent is replaced by the newborn child (i.e., $\underline{x}_s^{(k)} = \underline{u}_s^{(k)}$) if $\Psi\left[\underline{u}_s^{(k)}\right] < \Psi\left[\underline{x}_s^{(k)}\right]$;

- Step 4 – *Optimal Solution Update*: The current optimal solution is replaced (i.e., $\underline{x}_{opt}^{(k)} = \underline{u}_s^{(k)}$) if $\Psi\left[\underline{u}_s^{(k)}\right] < \Psi\left[\underline{x}_{opt}^{(k)}\right]$.

It is worth noting that the concept of iteration seems to disappear from the evolution process of the dynamic Differential Evolution since the population is continuously updated any time a better solution is found, even though the control level still determines the convergence of the procedure whether the iteration index, k, exceeds a pre-determined threshold, K_{max}.

2.2.3 Modified Differential Evolution

In order to obtain a good balance between exploitation and exploration ability, a modified Differential Evolution (MDE) was proposed in [36] to solve linear-array synthesis problems. On one hand, the convergence rate of the modified Differential Evolution has been enhanced by considering two modifications. First, the refinement mechanism for the best solution, $\underline{x}_{opt}^{(k)}$, is employed following the guidelines in [37]. The Fittest Individual Refinement (FIR) scheme with simplex crossover is adopted to generate more individuals belonging to the neighborhood of the optimal solution. Second and likewise for dynamic Differential Evolution [35], the new children are directly inserted into the current population if they outperform the corresponding parents. On the other hand, the modified Differential Evolution exploits a refreshing distribution operation to keep the diversity among the individuals of the population, thus avoiding premature convergence and the possibility of being trapped into local optima of the functional at hand. After a fixed number of iterations, K_{fresh}, the population is ranked according to the fitness values of the individuals, and the convectional Differential Evolution operators are applied to the $S/2$ individuals having better fitness, while the remaining individuals are reinitialized.

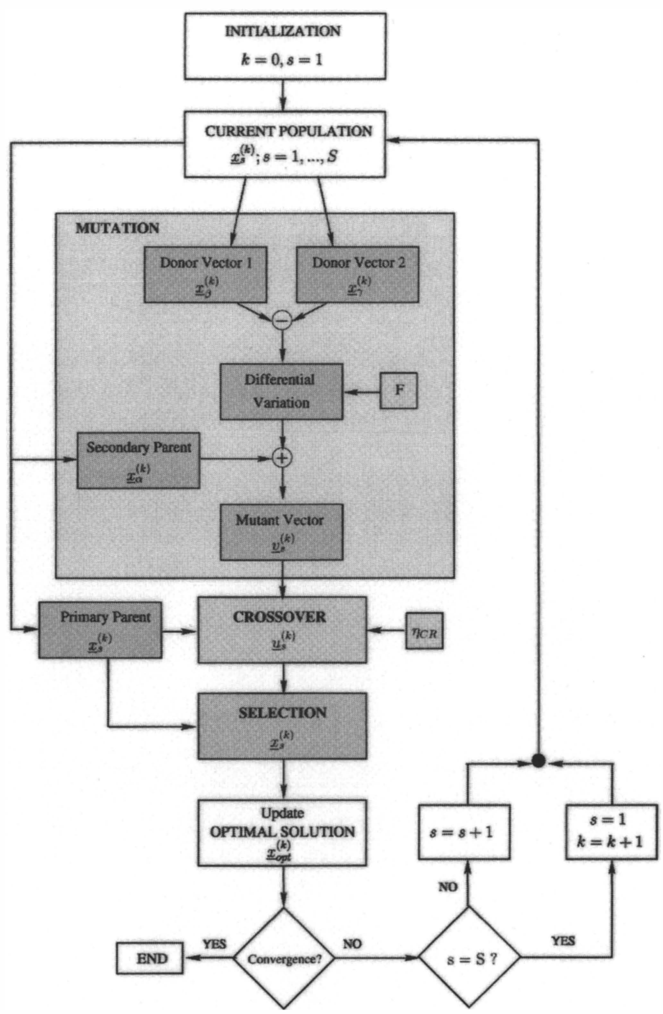

Figure 4. A dynamic Differential Evolution flowchart.

2.2.4 Hybrid-Coded Differential Evolution

In [38], a hybrid encoding of the unknown parameters was considered when dealing with the optimization of the difference pattern in monopulse antenna arrays. In more detail, a chromosome characterized by a set of N_1 integer values and a set of N_2 real values was optimized. To avoid coding and decoding operations, the crossover was properly customized to deal with the subset of N_1 integer parameters, also taking into account the fulfillment of the constraints of the antenna problem at hand:

$$u_{n,s}^{(k)} = \begin{cases} \left\lfloor v_{n,s}^{(k)} + 0.5 \right\rfloor & \text{if } r < \eta_{CR} \\ \left\lfloor x_{n,s}^{(k)} + 0.5 \right\rfloor & \text{otherwise} \end{cases}, \quad n = 1,\ldots,N_1, \tag{8}$$

where $\lfloor \cdot \rfloor$ identifies the integer part.

Unlike the implementation in [38], a two-step hybrid procedure was adopted in [24] to minimize the sidelobe level (*SLL*) in planar arrays. Towards this end, the problem was split into two simpler sub-problems, the first concerned with the optimization of real quantities (i.e., the inter-element spacing of the array), and the second one described with binary unknowns (i.e., the on/off elements of the array on a grid). Differential Evolution is used in the first step, while the second step considers a binary genetic algorithm.

2.2.5Differential Evolution with Best-of-Random Differential Mutation

Recently, an innovative mutation operator (i.e., the best-of-random (BoR) mutation) was proposed in [39] to provide a good tradeoff between search ability and guidance during the optimization process. The approach is similar to the DE/rand/1/* version, but here the secondary parent is selected as the fittest individual among three solutions randomly picked up from the population and used to generate the mutant vector. The other two individuals are used as donor vectors. The best-of-random works as follows:

$$\underline{v}_s^{(k)} = \underline{x}_\alpha^{(k)} + F\left[\underline{x}_\beta^{(k)} - \underline{x}_\gamma^{(k)}\right], \quad s = 1,\ldots,S, \tag{9}$$

where

$$\Psi\left[\underline{x}_\alpha^{(k)}\right] \le \min\left\{\Psi\left(\underline{x}_\beta^{(k)}\right), \Psi\left(\underline{x}_\gamma^{(k)}\right)\right\}, \tag{10}$$

and $\alpha \neq \beta \neq \gamma$. It is worth noting that the best-of-random mutation schema does not require additional control parameters.

2.2.6 Strategy Adaptation

In order to adapt the strategy to learn from previous generations, two versions of Differential Evolution, namely the DE/best/1/bin and the DE/rand/1/bin, were combined in [26]. The solution is looked for in a multi-minima functional by using the DE/best/1/bin to rapidly locate the attraction basin of a minimum. The DE/rand/1/bin is successively applied to avoid the trial solution being trapped in a local minimum.

3. Theoretical Background and Control Parameters

The efficiency of Differential Evolution has been proven in several problems [18, 34]. However, comprehensive parametric studies have shown that the behavior of Differential Evolution is greatly affected by differential-mutation-based strategies [26], and is very sensitive to the values of the control parameters [40]. Such events, as well as the difficulty to properly balance the exploration and exploitation of the method, have been confirmed by several works on this subject (see, for example, [40-45]).

On one hand, innovative operators exploiting geometrical relationships (e.g., trigonometric mutation [46]) have been introduced to improve the convergence speed. On the other, great care must be exercised to choose the control parameters of Differential Evolution. The crossover probability, η_{CR}, and the amplification coefficient, F, have to be carefully determined to avoid premature convergence to sub-optimal solutions, as well as slow convergence rates [28]. A basic rule suggests adapting the search step of the perturbation vectors, taking into account the diversity among the individuals of the population. The control parameters should allow large perturbations at the beginning of the evolution process, while small perturbations must apply at the end of the process, in the region of the solution space close to the attraction basin of the global optimum.

Since a general and useful (for whatever application) rule does not exist, some rules of thumb can be derived from the available literature [28, 40]. The dimension of the population should be chosen within $S \in [3 \times N, 8 \times N]$ [29, 41, 47, 48]. The values suggested for the scaling factor are $0.4 < F \le 1$ [47-49], and a good initial choice is $F = 0.6$ [41]. Values of the scaling factor larger than one (i.e., $F > 1$) allow escaping from local minima, but they decrease the convergence rate. As far as the probability of crossover is concerned, a good choice is $\eta_{CR} \in [0.3, 0.9]$ [41, 47]. Larger values of η_{CR} often speed up convergence.

4. Applications of Differential Evolution in Electromagnetics

This section is devoted to giving a comprehensive overview, to the best of the authors' knowledge, on the applications of Differential Evolution strategies to a variety of electromagnetic problems. In the following, three main macro areas in electromagnetic applications are considered: antenna (single element and array) synthesis and design, electromagnetic inverse scattering, and optimization of microwave components. For the sake of summary, Tables 1-3 also list the applications and related references discussed in the following.

4.1 Antennas

A large portion of the scientific literature on the application of Differential Evolution to antennas is concerned with the synthesis of arrays. In such a framework, Differential Evolution has been used to minimize functionals aimed at evaluating the mismatch between the synthesized field, $E(\theta, \phi, \underline{x})$, and the desired/reference field, $E^{ref}(\theta, \phi)$ [50, 51]:

Table 1a. A list of scientific publications on Differential Evolution as applied to antenna optimization.

Antennas			
Objective	**Subject**	**DE Strategy**	**Ref.**
SLL	Linear array	DE/best/1/bin	[50]
		De/best/1– DE/rand/1/–	[49]
		MDE	[36]
		DE/BoR/1/bin	[39]
	Planar array	Hybrid DE-GA	[24]
		–	[53]
	Reflectarray	DE/best/1/– DE/rand/1/–	[49]
	Thinned linear array	DE/best/1/bin	[47]
	Conformal array	DE/rand/1/bin	[23]
	TM linear array	DE/best/1/bin	[56]
		DE/best/1/bin	[57]
	TM planar array	DE/best/1/bin	[58]
		DE/best/1/bin	[59]
		–	[60]
	TM circular array	DE/best/1/bin	[61]

Table 1b. A list of scientific publications on Differential Evolution as applied to antenna optimization.

Antennas			
Objective	**Subject**	**DE Strategy**	**Ref.**
$SLL^{\Delta Mode}$	Monopulse linear array	Hybrid-coded DE	[38]
		MDE	[36]
$Directivity^{\Delta Mode}$	Monopulse linear array	Hybrid-coded DE	[55]
Pattern nulling	Linear array	DE/best/1/bin	[29]
	Planar array	DE/best/1/binwith jitter	[54]
Power pattern shaping	TM Linear array	DE/best/1/bin	[62]
	TM Semicircular array	DE/best/1/bin	[63]
Footprint pattern	TM Planar array	DE/best/1/bin	[64]
Pulse Doppler radar	TM Linear array	DE/best/1/bin	[65]
Multiple Patterns	TM Linear array	DE/best/1/bin	[66]
MC Compensation	TM Linear Array	DE/best/1/bin	[67]
Gain	Spherical Luneberg lens	–	[68]

Table 2. A list of scientific publications on Differential Evolution as applied to inverse scattering.

Inverse Scattering			
Material	**Objects**	**DE Strategy**	**Ref.**
PEC	2D Circular cylinders	DE/best/1/bin	[48]
PEC	2D Elliptical cylinders	DE/best/1/bin	[70]
PEC	2D Cylinders	DE/best/1/bin	[34]
PEC	2D Cylinders	GDES	[33]
PEC	2D Cylinders	DDE	[35]
PEC	2D Cylinders	DE/rand/1/bin	[18]
Dielectric	1D Scatters	DE/rand/1/bin	[72]
Dielectric	2D Buried structures	DE/rand/1/bin	[73]
Dielectric	2D Buried structures	DE/best/1/– DE/rand/1/–	[26]
Conductors	3D Buried spheroidal scatterers	–	[74]
Conductors	3D Buried spheroidal scatterers	GDES	[75]

Table 3. A list of scientific publications on Differential Evolution as applied to other electromagnetic topics.

Other Electromagnetic Problems		
Optimization Subject	**DE Strategy**	**Ref.**
Radial active magnetic bearings	–	[76]
UWB radio – system pulses	DE/rand/1/–	[77]
Electromagnetic property of composite materials	DE/best/1/bin	[78]
Microwave filters	Multi-objective DE	[79]

$$\Psi_1(\underline{x}) = \sqrt{\frac{\int_0^{2\pi}\int_0^{\pi/2}\left|E(\theta,\phi,\underline{x}) - E^{ref}(\theta,\phi)\right|^2 d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2}\left|E^{ref}(\theta,\phi)\right|^2 d\theta d\phi}}, \quad (11)$$

and the fulfillment of some pattern-performance criteria (e.g., sidelobe level, *SLL*; beamwidth, *BW*; and directivity, *D*) [23, 25, 38]:

$$\Psi_2(\underline{x}) = \sum_{p=1}^{P} w_p \Psi_p(\underline{x}), \quad (12)$$

where $\Psi_p^{ant}(\underline{x})$ is related to the *p*th pattern feature and w_p is a real and positive weighting coefficient.

A classic optimization problem when dealing with antenna arrays is the minimization of the sidelobe level. In [50], the cases of symmetric linear arrays having 30 and 48 elements, respectively, was treated. The definition of both element weights and positions was addressed by using cost functions similar to those in Equations (11) and (12), the latter aimed at achieving desired values of sidelobe level, bandwidth, and direction of the main beam. The control parameters of Differential Evolution were set to $S = 5N$, $F = 0.6$, and $\eta_{CR} = 0.9$. The optimization of $N = \{15,16,23\}$ real unknown parameters was performed in three different examples. Comparisons with a real-coded genetic algorithm were provided to assess the enhanced convergence rate of Differential Evolution.

The use of unequally spaced arrays has attracted great attention, since they achieve low sidelobe levels with a limited number of elements. Since the inter-element spacings are real quantities, Differential Evolution is intrinsically a suitable optimization tool for such synthesis problems [36, 39, 49]. Position-only and position-phase symmetric linear arrays, having 31, 32, and 60 elements, were considered in [49]. The values of the Differential Evolution control parameters were chosen in the range $0.5 \le \eta_{CR} \le 1$ and $0.4 \le F \le 1$. As far as the computational resources were concerned, the simulation ran for $K = 300$ iterations on an Intel Pentium-III PC with an 800 MHz processor. By considering $S = 320$ trial solutions, the simulation took about 43.7 minutes. Position-phase synthesis of uniform-amplitude arrays was also dealt with in [36] with modified Differential Evolution. Comparisons with conventional Differential Evolution have shown that modified Differential Evolution allows one to obtain the same sidelobe values, but with a faster convergence rate. More specifically, the same example in [49] was analyzed with modified Differential Evolution, and the number of cost-function evaluations was reduced by more than one-third, from 96000 iterations (i.e., $K = 300$ generations with a population of $S = 320$ individuals) down to 60000 iterations. More recently, the DDE/BoR/1/bin was also applied to this kind of problem, and an extensive set of numerical simulations was performed in [39]. The Differential Evolution parameters for the synthesis of a 32-element symmetric linear array were set to $S = 60$, $F = 0.4$, and $\eta_{CR} = 0.7$.

Other Differential Evolution implementations and strategies have been considered for the minimization of the sidelobe levels in planar arrays. The hybrid two-step approach [24] exploited Differential Evolution at the first step for optimizing the inter-element distances of a sparse linear array used as a building block for a planar-array architecture. A binary genetic algorithm was then applied to thin the filled configuration (replicating the sparse linear array obtained through Differential Evolution) for minimizing the sidelobe levels. As a representative test case, a 31-element linear array, symmetric with respect to the antenna's center, was optimized ($N = 15$) with Differential Evolution by setting $S = 100$ and $F = 0.5$ ($\eta_{CR} = 1$).

Inspired by [52], where a genetic-algorithm-based optimization tool was used, the rotation angles of the linearly polarized microstrip-patch antennas of an 8×8 planar array were determined by means of Differential Evolution to suppress the lobes outside the main-beam region [53].

Although the minimization of the sidelobe levels is sometimes enough to obtain suitable beam patterns for reducing the problems related to noise and interference, nulling is necessary in those cases where jammers or interference are characterized by high powers. In this framework, Differential Evolution has also provided good performance on a set of benchmark examples. More specifically, the optimization of the amplitude weights of a symmetric uniform linear array was dealt with in [29]. Differential Evolution ($S = 5N$, $F = 0.6$, and $\eta_{CR} = 0.9$) showed distinct advantages compared to modified touring ant colony optimization (MTACO) and the standard binary-coded genetic algorithm. The synthesis of planar arrays with prescribed nulls by means of position-only and position-amplitude optimization was carried out in [54] by means of the DE/best/1/bin with jitter [45]. An additional term was added to the cost function to penalize the solutions characterized by a minimum spacing between two adjacent elements below a fixed threshold. An example concerned with the optimization of a 36-element array with $S = 200$, $F = 0.2$, and $\eta_{CR} = 0.95$ was reported, as well.

Besides the optimization problems reported above, Differential Evolution has been also used in many other situations within the framework of array synthesis. In [49], the element positions of unequally-spaced reflectarrays were optimized to minimize the sidelobe levels, while considering a constraint on the minimum distance required between two close elements. In contrast, the optimization of some pattern features (e.g., sidelobe levels and directivity) of the difference mode in monopulse radar arrays was carried out through Differential Evolution [36, 38, 55]. To generate a compromise difference pattern with a low sidelobe level, an innovative hybrid real/integer-coded Differential Evolution was proposed in [38], by optimizing the aggregation of the elements into Q subarrays (unknowns) and their weights. Examples of symmetric linear arrays having 100 elements (i.e., $N = 54$ with 50 integer unknowns and $Q = 4$ real unknowns) and 20 elements (i.e., $N \in [12, 20]$ with 10 integer unknowns and $Q \in [2, 10]$ real unknowns) were considered with a Differential Evolution implementation characterized by $F = 0.5$, $\eta_{CR} = 0.7$, and $K_{max} = 1000$. The same approach was successively extended to the optimization of the directivity [55] by running a DE/rand/1/exp with $S = 10N$, $F \in [0.5, 2]$, $\eta_{CR} = 0.8$, and $K_{max} = 2000$. The mean computation time for each iteration when considering $N = 30$ (20 integer unknowns and $Q = 10$ real unknowns) was equal to 0.17 s on a 1.5 GHz PC with 512 MB of RAM. More recently, modified Differential Evolution [36] has been applied to the benchmark problems in [38]. As expected, results that were improved – both in terms of cost-function minimization and convergence rate – resulted, attesting to the efficiency and reliability of modified Differential Evolution.

In the last years, there has also been a growing interest in the synthesis of time-modulated (TM) arrays. In this framework, Yang and co-workers profitably applied Differential Evolution to several optimization problems. The joint minimization of the sidelobe levels and of the power losses generated by the periodic on-off commutations of the radio-frequency switches was considered. The static element (amplitude) excitations as well as the switch-on intervals were optimized in case of linear arrays [56, 57], planar arrays having circular/almost-circular boundaries [58, 59] or hexagonal shapes [60], and circular arrays [61]. Moreover, power-pattern synthesis problems, devoted to arbitrarily shaping the beam (e.g., a flat-top beam) in linear [62] as well as semicircular [63] time-modulated arrays were dealt with. In the latter case, both the amplitudes and phases of the complex static excitations were optimized by means of Differential Evolution. The DE/best/1/bin was used to generate footprint patterns from time-modulated planar arrays [64], and to synthesize time-modulated linear arrays suitable for airborne pulse-Doppler radars [65]. Moreover, effective Differential-Evolution-based procedures were proposed for generating multiple patterns [66], and to address mutual-coupling (MC) compensation problems [67].

Other antenna problems where Differential Evolution has been effectively used as an optimization tool are the synthesis of a uniform-amplitude thinned linear phased array [47], sidelobe minimization in conformal phased arrays [23], and the design of beamforming networks for scanned multibeam antenna arrays based on coherently radiating periodic structures [25].

Besides antenna arrays, single radiating elements have also been synthesized with Differential Evolution. A representative example is that discussed in [68], where a Luneberg lens antenna was designed by optimizing the focus distance, the size of the feed aperture, the layer thickness, and the dielectric constants of the shells.

4.2 Inverse Scattering

The reliability of Differential Evolution in dealing with multi-minima functionals, and the improved convergence rate as compared to genetic algorithms when applied to small-scale real-valued problems, are the main motivations for its use in electromagnetic inverse scattering [69]. The reconstruction of both penetrable objects as well as perfect electric conductors (PECs) has been carried out with Differential-Evolution-based approaches. In the former case, the image of the region under test (called the investigation domain, D_i) is obtained through the retrieval of the spatial distribution of the contrast function, $\underline{\tau}$, modeling the electromagnetic properties (i.e., permittivity, permeability, and conductivity) of D_i. To retrieve the unknown vector $\underline{\tau}$, the following cost function, proportional to the mismatch between measured and reconstructed fields, is minimized:

$$\Phi_1\left(\underline{\tau}, \underline{E}_{tot}\right) = w_S \frac{\left\| \underline{E}_{scatt} - \left[G_{EXT}\right] \underline{\tau} \underline{E}_{tot} \right\|^2}{\left\| \underline{E}_{scatt} \right\|^2} + w_D \frac{\left\| \underline{E}_{inc} - \underline{E}_{tot} + \left[G_{INT}\right] \underline{\tau} \underline{E}_{tot} \right\|^2}{\left\| \underline{E}_{inc} \right\|^2}, \quad (13)$$

where $\underline{E}_{inc}$ and $\underline{E}_{tot}$ are the incident and total field vectors, respectively; $\underline{E}_{scatt} \triangleq \underline{E}_{tot} - \underline{E}_{inc}$; and $\left[G_{EXT}\right]$ and $\left[G_{INT}\right]$ are the external and internal Green's operators. Moreover, w_S and w_D are real and positive regularization weights [69].

In contrast, only the external equation is processed during the optimization when dealing with PECs, since the field is null within the objects. Therefore,

$$\Phi\left(\underline{\gamma}\right) = \frac{\left\| \underline{E}_{scatt} - \left[G_{EXT}\right] \underline{J}\left(\underline{\gamma}\right) \right\|^2}{\left\| \underline{E}_{scatt} \right\|^2}, \quad (14)$$

where the unknown parameter vector, $\underline{\gamma}$, describes the contours of the scatterers where the current, $\underline{J}$, is present.

In more detail, Differential Evolution was first applied to PECs. In [48] and [70], the two-dimensional imaging of circular and elliptical cylindrical conductors or tunnels was performed. The DE/best/1/bin was used ($S \in [5N, 10N]$, $F \in [0.4, 1]$, and $\eta_{CR} \in [0, 1]$) to obtain information about the positions and radius (i.e., $N = 3$) of circular cylinders in [48]. The same approach was extended to deal with elliptical shapes in [70], solving a problem of dimension $N = 5$. In order to reconstruct PECs with arbitrary shapes, the surfaces of the scatterers have been approximated with cubic B-spline functions [34]. Likewise [48], the same version of Differential Evolution was adopted to optimize the control points of the spline functions, and a population of $S = 5N$ individuals was used with the following setup: $F = 0.7$ and $\eta_{CR} = 0.9$. The approach was compared in [34] with the real-coded genetic algorithm [71] on a set of benchmark examples. Moreover, the reconstruction of real data was also considered. The main drawback of the approach in [34] was that the reconstruction results strongly depended on the knowledge of the number of objects (needed a priori) that are supposed to lie in D_i. The Differential Evolution strategy with individuals in groups [33] was proposed to overcome this problem. As expected, it significantly outperformed conventional Differential Evolution [34], both in terms of convergence performance and reconstruction accuracy. To further improve the convergence rate, the use of the dynamic Differential Evolution (DDE) strategy was also investigated [35].

Recently, Differential Evolution has been compared with another evolutionary algorithm suitable for continuous optimization (i.e., the Particle Swarm Optimizer) on a set of representative examples when applied to the reconstruction of PEC cylinders [18] and one-dimensional dielectric scatterers [72]. In both cases, the

Differential Evolution control parameters were set to $F = 0.5$ and $\eta_{CR} = 0.8$.

The detection of two-dimensional buried inhomogeneities was carried out with conventional Differential Evolution in [73], and with a Differential-Evolution-based adaptive strategy in [26]. The extension to three-dimensional problems was dealt with in [74] and [75] by discussing the detection of unexploded ordnance (UXO) and lossy spherical objects buried in the subsoil, respectively. In this latter case, the Differential Evolution strategy with individuals in groups implementation was exploited by evolving in parallel multiple populations as in [33], and using a communication strategy between groups during the iterations.

4.3 Other Optimization Problems

Although the greatest part of the Differential Evolution literature is focused on optimization problems concerned with antenna synthesis and inverse scattering, Differential Evolution has been also applied to other topics in electromagnetics.

In [76], the optimization of radial active magnetic bearings was addressed. Dealing with ultra-wideband (UWB) radio systems, the DE/rand/1 scheme was used in [77] to optimize the source pulses and detection templates. Moreover, the analysis of the effective electromagnetic properties of composite materials with aligned non-spherical inclusions was carried out in [78].

Recently, a multi-objective version of Differential Evolution was presented in [79] for the design of multilayer dielectric microwave filters.

5. Conclusions

In this paper, a review of Differential Evolution as applied to electromagnetics has been presented. Beyond conventional Differential Evolution, modified and customized Differential Evolution versions have been described, to point out the main similarities and differences among the various implementations. Some theoretical hints on the influence of the setting of the parameters on the convergence behavior of the algorithm have been also given. The applicability of Differential-Evolution-based approaches to a broad class of optimization problems in electromagnetics has been illustrated by reporting, to the best of the authors' knowledge, the state-of-the-art on the subject.

Although it cannot be stated that Differential Evolution is better than other evolutionary algorithms (by virtue of the "no free lunch theorem" [80]), Differential Evolution generally outperforms other approaches dealing with small-scale optimization of continuous variables. Moreover, it shares several advantages with other population-based stochastic procedures, namely, the possibility of introducing physical constraints or a priori knowledge about the problem at hand in a simple way, hill-climbing features allowing escaping from local minima, differentiation of the functional is not required, and easy integration with gradient-based optimization tools.

As for optimization problems in electromagnetics, although they are generally characterized by a large number of unknown parameters – therefore limiting the potential application of conventional evolutionary-algorithm-based approaches – some parallel-processing architectures have been used to increase the computational efficiency of Differential Evolution. In this framework, data-decomposition schemes were proposed in [81-83], where the population is split into smaller sub-populations (similarly to what is done in the Differential Evolution strategy with individuals in groups), and each group of individuals is assigned to a different processor node. Suitable information-exchange procedures are then implemented to allow communication between all individuals and to guide the evolution process.

Differential Evolution therefore represents a reliable and effective alternative method to be carefully considered when approaching an optimization problem in electromagnetics, especially when dealing with floating-point unknowns. However, it is also worth emphasizing the need for future and further mathematical investigations on Differential-Evolution properties, and on its behavior during its iterative search for the optimal solution. As a matter of fact, Differential Evolution is still in its infancy. Analytical proofs about the convergence would enhance its overall performance, and also potentially reveal additional unexplored areas of application.

6. References

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